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OPTIMAL DESIGN OF SPIRAL CASING TONGUE AND WICKET GATE ANGLE BY DECOMPOSITION METHOD

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SUMMARY

A method of optimal design of Francis turbine tongue and wicket gate angle for given spiral casing is proposed. The potential flow in the doubly connected domain is decomposed into basic and circulation flows. The intensity of circulation is then calculated by the least-squares method minimizing the error function equal to the sum of squares of differences between given and calculated circumferential velocities in the outflow boundary nodes. It is shown that the error function has a sharp minimum, which qualifies the proposed method as well defined. For given numerical example, the variations in the outflow velocity angles are much smaller for optimal than for already used non-optimal design. A finite element method is used, with originally developed pre- and post-processor and frontal solver suited for personal computers.

KEY WORDS Finite elements Turbomachinery flow Optimal design Decomposition method Potential flow

1. INTRODUCTION

The role of the spiral casing of Francis turbine is to distribute the incoming water as evenly as possible, firstly to stay vane ring and then, together with wicket gate, to the turbine runner.

The flow of the water in Francis turbine is turbulent. The effects of flow curvature and viscous forces result in secondary flow which has been observed experimentally.[1] The existing computer packages for numerical modelling of turbulent flow enable the flow analysis in all parts of Francis turbine.[2,3] These calculations are very expensive because of the large CPU time. On the other hand, potential flow calculations, although the cheapest, are still used for turbine flow modelling.[4,5] The assumption that the spiral casing flow can be modelled by potential flow with satisfactory accuracy has been proved.[6,13,14] In Reference 8 it has been shown that the differences between the outflow velocity angles from spiral casing without stay vane ring and wicket gate, calculated with potential flow code and experimentally measured are small.

The potential flow in spiral casing of Francis turbine is completely defined by the tongue geometry. The tongue itself cuts the otherwise doubly connected domain of spiral casing, making it simply connected. It is known that the potential flow solutions in doubly connected domains are multivalued.[7] The question is: could this multivaluedness of the flow be used for optimal tongue design? Here it will be shown that the answer is positive. In two-dimensional flows around aerofoils the Kutta–Joukowsky condition determines the desired solution out of infinitely many possible ones. It is of fundamental importance to realize that the condition that determines the desired solution in multiconnected domains should have proper physical justification. In References 8–10, it was not the case. The principle of minimization of energy functional used there (References 8–10) does not take into account the influence of the runner.

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In the present work, it is assumed that the viscous effects, for optimal spiral casing and tongue design, are confined to boundary layers, so that the optimization can be done using potential flow assumption. The viscous effects can be taken into account afterwards by enlarging the spiral casing sections for boundary layer thickness.

2. MATHEMATICAL FORMULATION

The governing equations of the irrotational flow of ideal and incompressible fluid are

$$\operatorname{rot} \mathbf{v}=0 \quad \text{in } \Omega, \tag{1}$$

$$\operatorname{div} \mathbf{v}=0 \quad \text{in } \Omega, \tag{2}$$

$$\mathbf{v}\cdot\mathbf{n}=g \quad \text{on } \partial\Omega, \tag{3}$$

where $\mathbf{v}$ is the velocity field in bounded domain $\Omega \subset R^3$, $\mathbf{n}$ a unit vector of the outward normal of $\partial\Omega$, and g given function on $\partial\Omega$.

The domain defined by spiral casing without tongue is doubly connected, while with tongue is simply connected. Here the mathematical formulation for doubly connected domain is given. Due to the symmetry of the domain and boundary conditions of spiral casing, only one half of the domain (its upper half) is considered.

Introducing velocity potential ϕ, the problem (1)–(3), for doubly connected domain has the solution:[7,12]

$$\mathbf{v}=\operatorname{grad}\phi, \qquad \phi=\phi_0+\Gamma\phi_1, \tag{4}$$

where Γ is an arbitrary constant and potentials ϕ_0 and ϕ_1 are solutions of the following problems:

$$\Delta\phi_0=0 \quad \text{in } \Omega$$

$$\frac{\partial\phi_0}{\partial n}=g_1 \quad \text{on } S_1, \qquad \frac{\partial\phi_0}{\partial n}=g_2 \quad \text{on } S_2, \qquad \frac{\partial\phi_0}{\partial n}=0 \quad \text{on } S_3, \tag{5}$$

$$\Delta\phi_1=0 \quad \text{in } \Omega$$

$$\frac{\partial\phi_1}{\partial n}=0 \quad \text{on } \partial\Omega,$$

$$\phi_1|_{\Sigma^+}-\phi_1|_{\Sigma^-}=1, \qquad \left.\frac{\partial\phi_1}{\partial n}\right|_{\Sigma^+}-\left.\frac{\partial\phi_1}{\partial n}\right|_{\Sigma^-}=0. \tag{6}$$

Here S_1 and S_2 are inflow and outflow boundary of spiral casing, S_3 solid boundary, $\partial\Omega=S_1\cup S_2\cup S_3$, Σ cutting surface that makes domain simply connected $\Omega=\Omega\backslash\Sigma$, and Σ^+, Σ^-, its positive and negative sides, respectively (see Figure 1).

3. VARIATIONAL FORMULATION

The proposed method uses variational formulation of problems (5) and (6). Multiplication of Laplace equation $\Delta\phi_0=0$ by an arbitrary smooth-enough function ψ followed by partial integration leads, taking into account the boundary conditions, to the following variational formulation of problem (5), equivalent to equations (5): Find $\phi_0\in H^1(\Omega)$ satisfying

$$\int_\Omega \nabla\phi_0\cdot\nabla\psi \,\mathrm{d}\Omega=\int_{S_1} g_1\psi \,\mathrm{d}S+\int_{S_2} g_2\psi\,\mathrm{d}S, \quad \forall\psi\in H^1(\Omega), \tag{7}$$

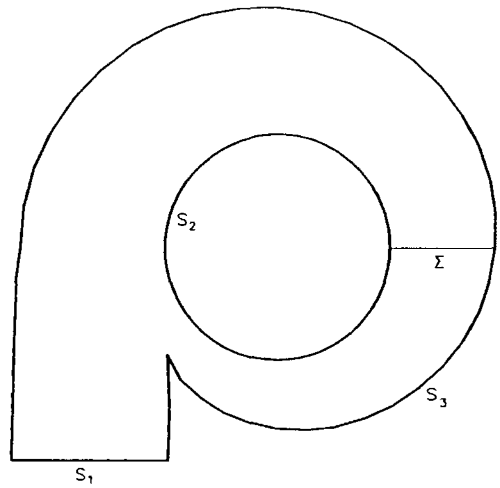

Figure 1. Spiral casing domain and its boundaries and cut

where $H^1(\Omega)$ is Soboljev function space—the space of functions which are, together with their first derivatives, square integrable. It is convenient to introduce a new function $\omega \in H^1(\Omega)$ for problem (6) with the following properties:

$$\phi_1 = \varphi + \omega, \qquad \frac{\partial \omega}{\partial n} = 0 \quad \text{on } \partial\Omega, \quad \omega|_{\Sigma^+} - \omega|_{\Sigma^-} = 1. \tag{8}$$

The variational formulation of problem (6) is then as follows: Find $\varphi \in H^1(\Omega)$ satisfying

$$\int_\Omega \nabla\varphi \cdot \nabla\psi \, d\Omega = -\int_\Omega \nabla\omega \cdot \nabla\psi \, d\Omega, \quad \forall \psi \in H^1(\Omega). \tag{9}$$

4. FINITE ELEMENT FORMULATION

The domain Ω is approximated by curved polygonal domain discretized by 20-nodal isoparametric finite elements, curved bricks. The solutions ϕ_0 and φ of problems (7) and (9) are approximated by linear combinations of nodal functions N_i of serendipity type:

$$\phi_0 = \sum_{i=1}^{M} \phi_0^i N_i, \qquad \varphi = \sum_{i=1}^{M} \varphi^i N_i, \tag{10}$$

where M is total number of nodes. The Galerkin method is accepted: for weighted functions of the variational problems (7) and (9), nodal functions N_i are chosen. The function ω is defined by the nodal functions of the nodes lying on the cut Σ and the characteristic function η^+ of the set Ω_{Σ^+}, equal to the union of finite elements lying on the Σ^+ side of the cut plane having at least one node in Σ^+:

$$\omega = \eta^+ N_\Sigma, \qquad N_\Sigma = \sum_{j=1}^{M_c} N_{pj}, \tag{11}$$

where M_c is total number of nodes in Σ, pj is global number of the jth node in Σ. Characteristic function η^+ is equal one in Ω_{Σ^+}, zero in $R^3 \backslash \Omega_{\Sigma^+}$. Note that ω is not in V_h, a finite-dimensional subspace of $H^1(\Omega)$. Finite element formulation of problems (7) and (9) gives the following systems of linear equations:

$$\mathbf{A}\phi_0 = \mathbf{b}_0, \qquad \mathbf{A}\varphi = \mathbf{b}_1, \tag{12}$$

where

$$A_{ij} = \int_\Omega \nabla N_i \cdot \nabla N_j \, d\Omega,$$

$$b_0^i = \int_{S_1} g_1 N_i \, dS + \int_{S_2} g_2 N_i \, dS,$$

$$b_1^i = \int_\Omega \nabla\omega \cdot \nabla N_i \, d\Omega, \quad i, j = 1, 2, \ldots, M. \tag{13}$$

Both systems in (12) have the same coefficient matrix which enables their solution in only one Gauss elimination procedure. Frontal method is used as best suited to personal-computer applications[15]

5. THE OPTIMAL SPIRAL CASING TONGUE DESIGN

The optimal shape of the tongue is one that gives the circumferential distribution of spiral casing outflow velocities as even as possible for given spiral casing geometry. In this case, the wicket gate losses will be the least for regular wicket gate cascade. The proposed method gives the stay vane mean lines as well as their positioning. They have to be aligned with the streamlines leaving the spiral casing at uniform circumferential spacing equal to $2\pi/z$ where z denotes the number of vanes. It should be pointed out that their shape, as obtained by this procedure, is not uniform around the axis. For the assumed values of wicket gate inflow angles α_i and given radial component of outflow velocity, calculated from the turbine flow rate, the circumferential component v_{ui} of the outflow velocity is calculated. The unknown circulation Γ_i is then calculated by the least-squares method as proposed in Reference 11. The sum of the squares of the differences between the calculated circumferential components of outflow velocities based on Γ_i at the nodes on outflow boundary and v_{ui} are given by

$$\text{SUMS}_i = \sum_{j=1}^{s} \lambda_j [(\nabla\phi_{0j} + \Gamma_i \nabla\phi_{1j}) \cdot \mathbf{t}_j - v_{ui}]^2, \tag{14}$$

where s is the number of the nodes on outflow boundary S_2, $\mathbf{t}_j$ is unit circumferential vector at these nodes, λ_j, $j = 1, 2, \ldots, s$ are weight factors.

Minimization of (14) gives the unknown circulation:

$$\Gamma_i = \frac{\sum_{j=1}^{s} \lambda_j (v_{ui} - \nabla\phi_{0j} \cdot \mathbf{t}_j) \cdot \nabla\phi_{1j} \cdot \mathbf{t}_j}{\sum_{j=1}^{s} \lambda_j (\nabla\phi_{1j} \cdot \mathbf{t}_j)^2}. \tag{15}$$

Here it is proposed to choose such a wicket gate inflow angle α_{opt}, called optimal, for which the value of the error function SUMS given by (14) is minimal. The function $\text{SUMS}_i = f(v_{ui})$ is shown by numerical calculation to have a sharp minimum which qualifies the proposed method as well defined (see Figure 5). The circulation calculatd by (15) for α_{opt} is denoted optimal, Γ_{opt}.

The optimal tongue design is one that follows the stream surface of the velocity field calculated by optimal circulation Γ_{opt} and given by

$$\mathbf{v} = \nabla\phi_0 + \Gamma_{opt}\nabla\phi_1. \quad (16)$$

In this case the flow will attack the wicket gate cascade, with geometrical inflow angle α_{opt}, as smoothly as possible.

6. NUMERICAL EXAMPLE

Special preprocessor suited for the discretization of the spiral casing domain of Francis turbine, with graphical representation of finite element mesh, is developed. The input variables to the mesh generator are (1) number of cross-sections with meridional planes in circumferential direction, (2) radii of their centres and (3) their radii (the cross-sections are mostly circular) with some other dimensions defining deviation of the cross-section from the circle, like the details of the Piguet geometry. The grouping of elements is also an input variable. The nodes of elements on the cross-section boundary are calculated automatically, given the number of elements in the radial and axial directions. The cross-section internal nodes are calculated via the principle of minimum sum of the lengths of all elements edges lying in a given cross-section. The generated mesh is given in Figure 2, where edges are drawn as straight lines, although the geometry of spiral casing is approximated by curved edges (to reduce drawing time).

For the chosen Francis turbine spiral casing, the velocity field is calculated for two geometries: one, given in Figure 3 with given tongue and simply connected domain, and the other without the tongue and doubly connected domain; see Figure 4. The tongue in Figure 4 is constructed by the proposed method along the streamline for optimal circulation.

The error function (14) is given in Figure 5 as a function of outflow angle α. Its minimum for optimal angle $\alpha_{opt}(=69°)$ is $10 \cdot 16\ \mathrm{m^2/s^2}$ which is small if we take into account that $v_{uopt} = 12{\cdot}2$ m/s, and the number of outflow boundary nodes $s = 60$. The number of unknowns in the given example is 1846.

Figure 6 shows that in the case of optimal tongue, the outflow angle is in the interval $\alpha_{opt} \pm 2°$ for all angles but for a small region around 360° and 342°, where the geometry should still be optimized by changing the cross-section radius and position. For non-optimal tongue which has been used in the construction of turbine, the angle of variation is greater.

The proposed procedure gives only tongue and stay vanes mean lines, considering these elements to be infinitely thin. Their finite thickness in real spiral casing must be modelled afterwards. The number of wicket gate vanes is not optimized.

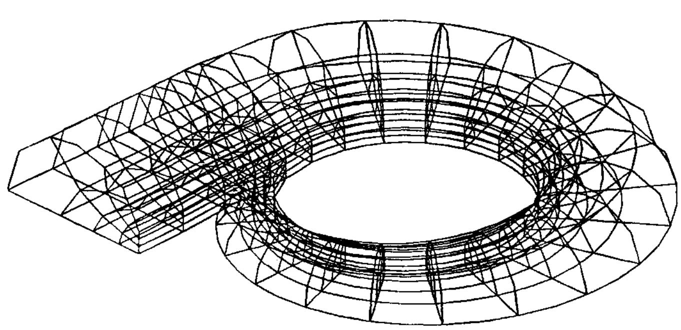

Figure 2. Finite element mesh

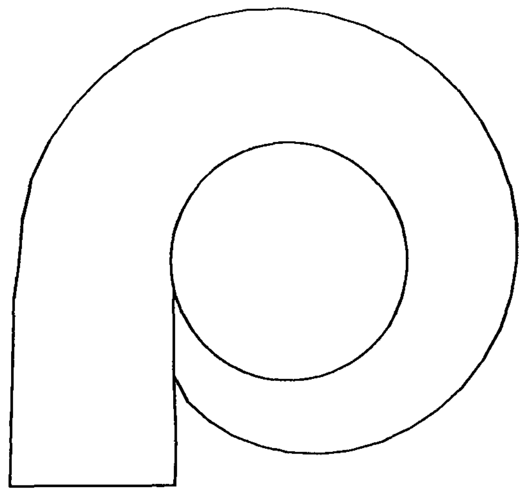

Figure 3. Spiral casing with non-optimum tongue

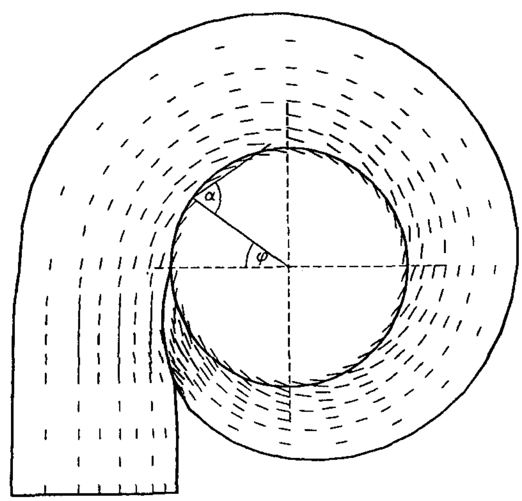

Figure 4. Spiral casing with optimum tongue

7. CONCLUSION

The method of optimal design of Francis turbine tongue and wicket gate angle for given spiral casing is proposed. A great simplicity of the proposed method lies in the fact that one can use the unknown circulation due to doubly connected domain and decomposition of the solution in the basic, ϕ_0, and circulation, ϕ_1, velocity potentials to solve the linear system with a lot of unknowns in only one sweep. By changing the circulation intensity Γ and superimposing both velocity fields, it is possible to calculate the optimal flow field without additional solutions of large systems minimizing error function, equal to the sum of the square of the differences.

It is proposed next to make the new mesh generator for the simply connected domain with spiral casing, optimal tongue and wicket gate at optimal inflow angle, designed by the proposed

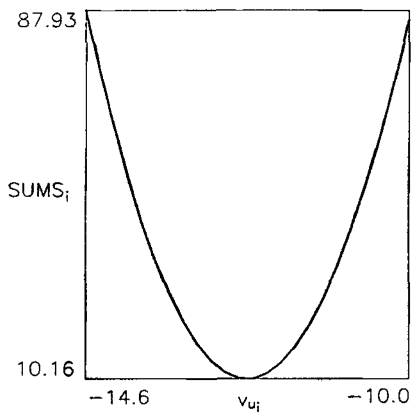

Figure 5. Error function

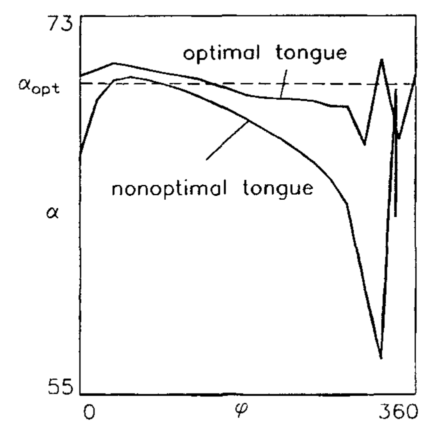

Figure 6. Outflow velocity angle α versus cross-section angle φ for optimal and non-optimal tongue

method. The flow field calculation in such an extended domain will be simple, without optimization, and near optimum. The viscosity effects for confined thin boundary layers can be allowed for with thick vanes for boundary layer thickness calculated from 2D boundary layer calculations.

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